Adaptive Transmission Schemes for Free-Space Optical Channel

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Abstract—Scintillation can be a major issue for free space optical communication (FSO) links even under clear sky condition. In this paper, adaptive techniques are proposed as a means to combat the scintillation. We first consider adaptive symbol-rate in which the symbol duration varies according to the channel condition. Joint adaptation of both the symbol-rate and power is then investigated. The adaptation algorithm in each case is formed as an optimization problem with the objective of maximizing the throughput of the system. The performance of the system with a bandwidth constrained detector is also evaluated. Results showed that significant improvement over the fixed transmission system can be obtained.

Keywords—Free-space optical communication, Scintillation, fading atmospheric channels, Adaptive-rate, Adaptive-power, Optimization.

I. INTRODUCTION

Optical fiber communication is an efficient method of transmitting data between two points through the use of the optical fiber cables. However, this technology can not be used in areas where a physical connection is not feasible and alternative methods have to be utilized. Radio frequency transmission can be used but the spectrum is very congested and obtaining a license for a service band is increasingly difficult. Unlike RF transmission, free space optical communication (FSO) possesses a huge and unlicensed spectrum which can be utilized to offer very high speeds of wireless data exchange. Therefore, it can serve as an excellent alternative and/or complimentary mean to meet the rapidly increasing demand for data services. FSO also has many other advantages over RF transmission, such as the high security, more compact equipment size and weight and immunity to interference and jamming [1].

One of the most challenging issues of the (FSO) communication is its susceptibility to atmospheric effects. Absorption and scattering of the light due to the interaction with the constituent gases and particles of the atmosphere introduces a wavelength dependent attenuation. The attenuation is notably low at several transmission windows within the wavelength range from 800nm to 1600nm. One of these windows, namely the 1550nm, is preferable since it is relatively “eye safe”. It also happens to be the best window for fiber-optic communication, therefore, these reliable and commercially-available devices can be utilized for FSO. Weather conditions such as fog, rain, snow and clouds can cause strong attenuation which might result in a complete transmission failure [2], [3], [4]. The transmit power can be increased to substitute these losses but the maximum allowable power is subject to eye safety standards. A hybrid FSO/RF link was proposed to overcome the problem of attenuation and increase the reliability of transmission at the various conditions of the atmosphere [5], [6].

In addition to the atmospheric attenuation, received light experiences random fluctuations in amplitude and phase due to the inhomogeneity of the refractive index of the atmosphere in a phenomenon known as atmospheric turbulence or scintillation. The variation of the refractive index is caused by variation in the air density of the atmosphere mainly due to temperature and pressure gradients [1]. Atmospheric turbulence is a major challenge even under clear sky conditions and has similar effect as the multipath fading experienced in RF transmission. The performance of the transmission dramatically decreases due to the scintillation when no fading mitigation is used [7], [8].

Various mitigation methods have been proposed in literature, which include the use of coding, diversity transmission techniques and adaptive optics [9], [10], [11]. Adaptive transmission techniques where transmission parameters are varied according to the instantaneous condition of the channel were also proposed as efficient ways to improve the performance [12], [13], [14]. The coherence time of the free space optical channel ranges from several to tens of milliseconds [15], [10], [16]. However, these milliseconds values of coherence times are quite long when compared to typical symbol rates of the FSO systems and hence the channel is modeled as a slow fading process. Moreover, the FSO channel can have a high degree of reciprocity in the axially-aligned bidirectional systems, i.e. the fading level at each end of the system can be highly correlated [16], [17]. These properties can be utilized to obtain instantaneous knowledge of the channel state information (CSI) at the transmitter and discard the necessity and complexity of a separate feedback channel. Therefore, it is reasonable to assume CSI at both the the transmitter and receiver, as required for adaptive transmission.

While the FSO channel bandwidth is large, communication rate is usually limited by the response time (response bandwidth) of the photodiode at the receiver. In optical communication systems, a photodiode with a large detection area and large bandwidth would be preferred. However, these requirements are in contradiction to each other. Large bandwidths are obtainable in devices with small active detection areas. For example, commercial photodiodes with active area of φ200μm have bandwidths ranging from several hundreds of MHz up to approximately one GHz [18], [19]. The bandwidth will be reduced to around one hundred MHz when an active area

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of $\phi3mm$ is required. The limited frequency response of the photodiode can impact the transmission and has to be taken into account in evaluating the system performance.

In this paper, two adaptive schemes are considered: adaptive symbol-rate and jointly adaptive power and symbol-rate. This investigation extends the analysis in [12]. It takes into account the optimization of the number and values of the symbols. The results are presented in terms of the relative throughput compared to a fixed transmission rate for clear understanding of the performance gain. A closed form representation for the adaptive throughputs is derived for the adaptive symbol-rate scheme. Moreover, the impact of having ISI due to the receiver’s limited bandwidth is also addressed.

The organization of the paper is as follows. The system model and the throughput performance of the fixed transmission scheme are presented in section II. The performance of both adaptive schemes with and without the impact of detector’s frequency-limited response are listed in sections III and IV respectively. A conclusion is provided in the last section.

II. SYSTEM MODEL

In this work, OOK intensity modulation and direct detection is considered. The input data for both the fixed and adaptive systems is assumed to be coded by a regular LDPC coder. The code, in the case of fixed transmission, is assumed to utilize a large interleaver to avoid burst errors. The general model for the free space optical communication system used in this paper is shown in figure 1.

![Fig. 1. A general model for the adaptive free space optical communication system](image)

In figure 1, $x$ is the transmitted symbol value, $T$ is the pulse width and $P$ is the optical transmission power. $n_{ele}$ and $n_{op}$ represent the electrical and optical noise sources respectively. $g(t)$ is the transmitted pulse shape and $c(t)$ is the photodiode impulse response. The factor $h$ represents the fading channel and is assumed to follow a lognormal distribution with scintillation index of $SI = \sigma_f^2 = E\{h^2\}/E\{h\}^2 - 1$. An aggregate model of the FSO channel must include both the atmospheric attenuation and scintillation. However, since the attenuation varies on a long-term basis and since this paper addresses the issue of scintillation, the lognormal model will be sufficient.

The probability density function (pdf) of the lognormal distribution is closely related to that of the normal distribution and it is given by [20]

$$f_h(h) = \begin{cases} \frac{1}{h\sqrt{2\pi}\sigma} \exp\left(-\frac{(\log(h) - \mu)^2}{2\sigma^2}\right), & \text{for } h \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where $\mu$ and $\sigma^2$ are the mean and variance of the related normally distributed random variable.

For a given scintillation index $\sigma_f^2$, $\mu$ and $\sigma^2$ can be calculated using the following equations:

$$\sigma^2 = \log(1 + \sigma_f^2) \quad \text{and} \quad \mu = -\frac{1}{2} \log(1 + \sigma_f^2) \quad (2)$$

We first consider a fixed transmission scheme where the symbol-rate $T_f$ and power $P$ are constant regardless of the instantaneous channel condition. When a photodiode with unlimited bandwidth is assumed, $c(t)$ will be a dirac delta function ($c(t) = \delta(h)$) and will have no impact on the received pulse. The output current of the fixed transmission system, assuming that the photodiode has a responsivity of 1, is given by

$$y(t) = xPhg(t) + n_{ele} + n_{op} \quad (3)$$

For simplicity and analytical tractability we assume the optical noise to be zero $n_{op} = 0$ and the electrical noise to be AWGN with a variance of $\sigma_n^2$. After matched filtering and sampling, the instantaneous signal to noise ratio of the system at the receiver output can be written as

$$\gamma_f = \frac{E\{x^2\}P^2h^2}{\sigma_n^2} \quad (4)$$

The noise power within the signal bandwidth can be represented by $\frac{N_0}{T_f}$, where $N_0$ is the noise power spectral density. The SNR can be, therefore, rewritten as

$$\gamma_f = \frac{4P^2h^2T_f}{N_0} \quad (5)$$

Since sufficiently large interleaver is assumed, the average SNR of the system can be calculated as follows:

$$\bar{\gamma_f}(SI) = \frac{4P^2E\{h^2\}T_f}{N_0} = \frac{4P^2(1 + SI)T_f}{N_0} \quad (6)$$

For a given scintillation strength, the transmission power and $T_f$ need to be high enough to provide a sufficient average SNR for reliable recovery. Figure 2 shows the BER performance versus SNR of the fixed system utilizing a regular LDPC code with degree 3 nodes under various scintillation strengths. The block length of the code is 5000 with code rate of 0.8.

For each case, a sharp waterfall error performance can be seen. Therefore, we make the assumption that the code will achieve error-free communication after a certain threshold and select the point with BER of $10^{-3}$ to be the threshold. The threshold is a function of the scintillation index and in this paper, the notation $\gamma_c(SI)$ will be used to represent this threshold. The values of $\gamma_c(SI)$ for the curves in figure 2 are listed in table 1.

<table>
<thead>
<tr>
<th>$SI$</th>
<th>$\gamma_c(SI)$ in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.8</td>
</tr>
<tr>
<td>0.5</td>
<td>10.7</td>
</tr>
<tr>
<td>1</td>
<td>14.6</td>
</tr>
<tr>
<td>2</td>
<td>19.65</td>
</tr>
</tbody>
</table>

Table 1. SNR threshold required by LDPC for reliable recovery for various values of SI.
The maximum performance (throughput) of the fixed scheme can be found from (6) by calculating $\frac{1}{T_f}$ at the corresponding SNR threshold. For a given transmission power and scintillation index, the throughput $R_f$ will be

$$R_f = \frac{1}{T_f} = \frac{4P^2(1 + SI)}{\gamma_c(SI)N_0}$$  \hspace{1cm} (7)

III. ADAPTIVE TRANSMISSION SYSTEM

For adaptive transmission, two schemes are considered: adaptive symbol-rate and adaptive power and symbol-rate. In both schemes, perfect channel state information (CSI) is assumed to be available at both the transmitter and receiver.

A. Adaptive symbol-rate scheme

In this scheme, the symbol rate $T$ is varied according to the channel condition $h$. The optimum scheme will select the minimum symbol rate to maximize the throughput while ensuring that the instantaneous SNR is equal or larger to the LDPC SNR threshold of the fixed and non-fading system $\gamma_c(0)$. Therefore, the optimum performance is found by solving the following optimization problem:

$$\text{maximize } T(h) \int \frac{1}{f_h(h)} \{1 + \frac{4P^2h^2N_0}{T(h)} \geq \gamma_c(0)\} f_h(h) dh$$  \hspace{1cm} (8)

where $f_h(h)$ is the probability density function of the Lognormal channel and the indicator function $1\{1\}$ returns 1 when the inequality inside the brackets holds true and 0 otherwise.

The optimum scheme selects the pulse width to be just sufficient to satisfy the equality in the SNR condition, i.e. $T(h) = \frac{\gamma_c(0)N_0}{4P^2h^2}$. The optimum throughput, therefore, becomes

$$R_T = \frac{4P^2h^2}{N_0\gamma_c(0)} f_h(h) dh$$  \hspace{1cm} (9)

To compute the relative gain in performance of the adaptive system compared to the fixed transmission, the throughput is normalized by that of the fixed system in (7). The relative throughput is given by

$$r_T = \frac{R_T}{R_f} = \frac{1}{T_f} \int \frac{\gamma_c(SI)}{\gamma_c(0)(1 + SI)} h^2 f_h(h) dh$$  \hspace{1cm} (10)

where

$$r_T = \frac{\gamma_c(SI)}{\gamma_c(0)} E\{h^2\} = \frac{\gamma_c(SI)}{\gamma_c(0)}$$  \hspace{1cm} (11)

The optimization problem in (8) is for continuously variable $T$ which is not suitable for practical implementation. In practice, it is more convenient to handle a finite set of rates instead of the continuously variable rate. Under the assumption of the finite set of rates $T_i \in \{T_1, ..., T_m\}$ where $T_0 = \infty > T_1 > \ldots > T_m > T_{m+1} = 0$, the optimal adaptation rule satisfies

$$T(h) = T_i, h \in [h_i, h_{i+1})$$  \hspace{1cm} (12)

where the infinite duration of $T_0$ represents no transmission when $h$ is very small. The optimization problem becomes

$$\text{maximize } T_i \sum_{i=1}^{m} \frac{1}{T_i} \int_{h_i}^{h_{i+1}} f_h(h) dh$$  \hspace{1cm} (13)

where $h_i = \sqrt{\frac{\gamma_c(0)N_0}{4P^2T_i}}$

The problem can also be written as a function of the lognormal cumulative distribution function $F_h$ as follows:

$$\text{maximize } T_i \sum_{i=1}^{m} \frac{1}{T_i} \int_{h_i}^{h_{i+1}} f_h(h) dh$$  \hspace{1cm} (14)

where

$$F_h(h) = 0.5 + 0.5\text{erf}\left(\frac{\log(h) - \mu}{\sqrt{2\sigma^2}}\right)$$

The relative throughput is given by:

$$\text{maximize } \sum_{i=1}^{m} \frac{T_i}{T_f} \left[F_h\left(\sqrt{\frac{\gamma_c(0)N_0}{4P^2T_i+1}}\right) - F_h\left(\sqrt{\frac{\gamma_c(0)N_0}{4P^2T_i}}\right)\right]$$  \hspace{1cm} (15)

Assuming $k_i = T_i/T_f$, the relative throughput can be rearranged as follows:

$$r_T = \max_{0 < k_1 < \ldots < k_m} \sum_{i=1}^{m} k_i \left[F_h\left(\sqrt{Xk_{i+1}}\right) - F_h\left(\sqrt{Xk_i}\right)\right]$$  \hspace{1cm} (16)

where

$$X = \frac{\gamma_c(0)(SI + 1)}{\gamma_c(SI)}$$

Taking the derivative w.r.t $k_i$ and equating to zero gives the following

$$\frac{dr_T}{dk_i} = \frac{0.5}{\sqrt{2\pi\sigma^2}} \left[\frac{k_{i+1}}{k_i} - 1\right] \exp\left(-\frac{\left(\log(\sqrt{Xk_i} - \mu)^2}{2\sigma^2}\right)ight)$$

$$+ 0.5 \left[\text{erf}\left(\frac{\log(\sqrt{Xk_{i+1}}) - \mu}{\sqrt{2\sigma^2}}\right) - \text{erf}\left(\frac{\log(\sqrt{Xk_i}) - \mu}{\sqrt{2\sigma^2}}\right)\right] = 0$$  \hspace{1cm} (17)
The solution of (17) gives the optimum symbol rates and consequently the relative throughput. It can be seen that the derivative w.r.t any $k_i$ depends only on that $k_i$, the previous one $k_{i-1}$ and the next one $k_{i+1}$. In addition, since $k_i \propto \frac{1}{n}$, the values of $k_0$ and $k_{m+1}$ are 0 and $\infty$ respectively. Therefore, solving for all the values in (17) requires a simple single direction search.

The analysis showed, in general, that significant improvements over the fixed transmission can be obtained. As expected, the performance gain of the adaptive scheme grows with the increase of the scintillation strength. The results of the relative throughput are listed in Table I for various values of $SI$ and $m$. It can also be seen that higher $m$ leads to better system performance, which is reasonable since there will be more options to select an instantaneous $T$ at any given channel condition.

**TABLE II. RELATIVE THROUGHPUT OF THE ADAPTIVE SYSTEM FOR VARIOUS VALUES OF SI AND M.**

<table>
<thead>
<tr>
<th>SI</th>
<th>Relative throughput</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m = 2$</td>
</tr>
<tr>
<td>SI=0.5</td>
<td>1.6</td>
</tr>
<tr>
<td>SI=1</td>
<td>3.32</td>
</tr>
<tr>
<td>SI=2</td>
<td>9.05</td>
</tr>
</tbody>
</table>

![Fig. 3. Values and probability of transmission of the individual optimized symbols for the case $m = 7$ and $SI = 1$.](image)

The numerical results also suggest that the individual values of the optimum pulse widths are roughly octave-spaced. This can be seen in figure 3 for the case of $m = 7$ and $SI = 1$. This property can be of advantage, since it is practically easier to implement a system with defined spacings between the individual symbols. The optimization problem in (16) for a system using octave-spaced symbol rates, i.e. $k_i \in \{k, 2k, 3k, \ldots, \frac{k}{2m}\}$, can be written as follows:

$$\max_k \sum_{i=1}^{m} k_i \left[ F_h \left( \frac{X_k}{2} \right) - F_h \left( \frac{X_k}{2} - 1 \right) \right]$$

The problem in (18) is a single variable optimization problem and can be easily solved. The relative throughput for the system with octave-spaced symbols is compared with that of all optimized symbols in Table 2.

**TABLE III. RELATIVE THROUGHPUT FOR ALL OPTIMIZED SYMBOLS AND OCTAVE-SPACED SYMBOLS**

<table>
<thead>
<tr>
<th>SI</th>
<th>Relative throughput</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m = 2$</td>
</tr>
<tr>
<td>SI=1 All optimized</td>
<td>3.32</td>
</tr>
<tr>
<td>Octave-spaced</td>
<td>2.99</td>
</tr>
<tr>
<td>SI=2 All optimized</td>
<td>9.05</td>
</tr>
<tr>
<td>Octave-spaced</td>
<td>7.93</td>
</tr>
</tbody>
</table>

Noticeable differences in the performance can be seen for both $SI = 1$ and $SI = 2$ when $m$ is small. However, the differences fade away as $m$ increases.

**B. Adaptive power and symbol-rate scheme**

In this scheme, both the power $P$ and the symbol rate $T$ are varied according to the channel condition to further improve the performance. The power variation is subject to both a peak power $P_{peak}$ and average power $P_{av}$ constraints. The optimum scheme can be also formulated as an optimization problem as follows:

$$\max P(h), T(h)$$

$$\int_0^\infty \frac{1}{T(h)} \frac{\gamma_c(h)}{N_0} f_h(h) dh \geq \gamma_c(0)$$

subject to

$$\int_0^\infty P(h) f_h(h) dh \leq P_{av}$$

and

$$P(h) \leq P_{peak}$$

The optimum scheme allocates just enough power and symbol-rate to satisfy the required SNR. Therefore, the equality in the LDPC condition can be used, i.e. $P(h) = \sqrt{\frac{\gamma_c(0) N_0}{4 h^2 T(h)^2}}$

The optimization problem can be rewritten as:

$$\max T(h)$$

$$\int_0^\infty \frac{1}{T(h)} f_h(h) dh$$

subject to

$$\int_0^\infty \frac{1}{\sqrt{T(h)}} f_h(h) dh \leq \sqrt{\frac{4 P_{av}^2}{\gamma_c(0) N_0}}$$

and

$$\sqrt{\frac{\gamma_c(0) N_0}{4 h^2 T(h)^2}} \leq P_{peak}$$

Following [12], it can be shown that the optimal solution $T(h)$ is a non-increasing function of $h$. Therefore, the relative throughput of the adaptive power and symbol-rate system under the assumption of finite set of symbol rates can be given...
by

\[
\text{maximize } \sum_{i=1}^{m} k_i \int_{h_i}^{h_{i+1}} f_h(h) dh \\
\text{subject to } \sum_{i=1}^{m} \sqrt{k_i} \int_{h_i}^{h_{i+1}} f_h(h) dh \leq \sqrt{\frac{\gamma_c(SI)}{\gamma_c(0)(SI + 1)}} \]

and

\[
\sqrt{\frac{\gamma_c(0)k_i(SI + 1)}{\gamma_c(SI)(\text{PAPR})^2}} \leq h_i
\]

where PAPR is the peak to average power ratio, i.e. \( p_{\text{peak}} = \text{PAPR} \times P_{\text{av}} \).

Matlab optimization toolbox [21] was used to solve the problem and find the optimum parameters and consequently the throughput. A non-linear and constrained optimization method, namely Karush-Kuhn-Tucker (KKT) method, was also used, involving numerical and analytical steps, to confirm several test points.

The results for various values of scintillation strengths and various values of (PAPR) are shown in table 4. The performance improvements can be seen compared to the case of the adaptive symbol-rate only (PAPR = 0). The throughput of the system increases as the condition of the peak power is relaxed since the system will have higher flexibility to optimally distribute the available power. However, the system peak power is subject to laser’s lifetime considerations.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
PAPR in dB & \( SI = 0.5 \) & \( SI = 1 \) & \( SI = 2 \) \\
\hline
0 & 1.6007 & 3.3197 & 9.0519 \\
3 & 6.3727 & 13.2159 & 36.0363 \\
6 & 25.0698 & 52.6135 & 143.0647 \\
\hline
\end{tabular}
\caption{Relative throughput of the adaptive power and symbol-rate scheme}
\end{table}

IV. EFFECTS OF A BANDLIMITED RECEIVER

In the previous section, the receiver is assumed to have unlimited bandwidth and it can accommodate any symbol-rate. A receiver with limited frequency response can lead to intersymbol interference (ISI) when the signal has wide bandwidth (short pulses). The symbol energy might be reduced even if ISI was removed since a portion of the signal will be filtered by the receiver. Assuming all ISI is removed, the signal current part at the receiver for a bandlimited receiver is given by

\[
s(t) = xP h[g(t) \ast c(t)]
\]

To evaluate the performance, the photodiode is assumed to have a first order low-pass response. In this case, \( c(t) \) is given by

\[
c(t) = \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right)
\]

where \( \tau \) is the filter time constant and is related to the filter cutoff frequency by \( f_c = \frac{1}{2\pi\tau} \). In this case

\[
s(t) = \begin{cases} 
    xP h(1 - \exp(-\frac{t}{\tau})) & \text{for } t \leq T \\
    xP h[\exp\left(\frac{T}{\tau}\right) - 1]^2 \exp(-\frac{t}{\tau}) & \text{for } t > T
\end{cases}
\]

The instantaneous signal to noise ratio can be found as follows:

\[
\gamma_b = \frac{\int_{0}^{\infty} s(t)^2 dt}{\sigma_n^2} = \frac{2T \int_{0}^{\infty} s(t)^2 dt}{N_0} = \frac{2E\{x^2\} P^2 h^2 A(T) T}{N_0}
\]

where

\[
A(T) = 1 + \frac{2\tau}{T} \left[\exp\left(-\frac{T}{\tau}\right) - 1\right] - \frac{\tau}{2T} \left[\exp\left(-\frac{2T}{\tau}\right) - 1\right] + \left[\frac{\tau}{2T} \left(\exp\left(-\frac{T}{\tau}\right) - 1\right)^2 \exp(-\frac{2T}{\tau})\right]
\]

The factor \( A(T) \leq 1 \) represents the reduction in the total signal energy due to the filtering process and it is a function of the symbol-rate \( T \) and the time constant \( \tau \) of the photodiode’s response. The average SNR can be calculated by averaging over the channel fading factor \( h \) and it is given by

\[
\bar{\gamma}(SI) = \frac{4P^2 E\{h^2\} A(T) T}{N_0} = \frac{4P^2(1 + SI) A(T) T}{N_0}
\]

For a fixed transmission system \( T = T_f \), the symbol rate needs to be selected wide enough to provide the LDPC SNR threshold, listed in table 1 for the utilized code, taking into account the attenuation \( A(T_f) \). The throughput of the system can be written as follows:

\[
R_{bf} = \frac{1}{T_f} = \frac{4P^2(1 + SI) A(T_f)}{\gamma_c(SI) N_0}
\]

A. Adaptive symbol-rate scheme

In the adaptive scheme, the attenuation varies with the instantaneous value of \( T \), which in turn depends on the channel condition. By applying the same procedures of section III, the throughput of the adaptive symbol-rate system with a finite set of rates can be written as

\[
\text{maximize } \sum_{i=1}^{m} \frac{1}{T_i} \left[ F_{h} \left( \sqrt{\frac{\gamma_c(0) N_0}{4P^2 T_{i+1} A(T_{i+1})}} \right) 
- F_{h} \left( \sqrt{\frac{\gamma_c(0) N_0}{4P^2 T_i A(T_i)}} \right) \right]
\]

The throughput results for both the adaptive and fixed schemes for a receiver utilizing a bandlimited photodiode (cutoff frequency of 1GHz) are shown in figure 4. The actual throughput is plotted against the quantity \( \frac{N_0}{N_c} \).

It can be seen that the gain from the adaptive scheme is similar to that of the bandwidth unlimited receiver when the power is low. For low power, the adaptive system selects wide symbol-rates to satisfy the SNR threshold and, therefore, the impact of the ISI will not be significant. However, the gain
is reduced as the power increases since high powers enable the fixed system to operate with short pulses with bandwidth comparable to that of the photodiode response. In this case, the adaptive scheme will have small room to vary the symbol-rate since it will face more severe ISI when trying to select shorter pulses. This can be seen more clearly in figure 5 where results of the relative throughput (throughput normalized by that of the fixed transmission scheme $R_{0,f}$) of the adaptive system and bandlimited photodiode with cutoff frequencies of 1GHz and 50 MHz are shown. The gain in the case of 50 MHz photodiode is slightly lower than that of the unlimited receiver at $P^2/N_0 = 0$dB. The gain drops with the increase of $P^2/N_0$ and asymptotically reaches a constant larger-than-one value. In the case of 1 GHz photodiode, the drop in the gain starts from $P^2/N_0 \approx 10$dB and also asymptotically reaches a constant value.

In practice, equalization can not perfectly remove the ISI without enhancing the noise. Therefore, the assumption of the ideal ISI removal will represent the upper limit of the performance. For this reason, the performance when practical linear equalizers are used is also evaluated. Zero-forcing and minimum mean square error (MMSE) equalizers are considered. The performance of these equalizers are evaluated in [22, chapter 9]. The relative throughput of the system when utilizing a linear equalizer is also shown in figure 5.

B. Adaptive power and symbol-rate scheme

Taking the attenuation into account, the optimization problem of the throughput for adaptive power and continuously variable symbol rate can be written as

$$\begin{align*}
\text{maximize} & \quad \int_0^\infty \frac{1}{T(h)} f_h(h)dh \\
\text{subject to} & \quad \int_0^\infty \sqrt{\gamma} \frac{f_h(h)}{\bar{T}(h)} dh \leq \sqrt{\frac{4P^2_{\text{av}}}{\gamma(0)N_0}} \\
\text{and} & \quad P(h) \leq P_{\text{peak}}
\end{align*}$$

(30)

The optimum scheme allocates just sufficient power to satisfy the threshold, therefore, the problem becomes

$$\begin{align*}
\text{maximize} & \quad \int_0^\infty \frac{1}{T(h)} f_h(h)dh \\
\text{subject to} & \quad \int_0^\infty \sqrt{\gamma} \frac{f_h(h)}{\bar{T}(h)} dh \leq \sqrt{\frac{4P^2_{\text{av}}}{\gamma(0)N_0}} \\
\text{and} & \quad \sqrt{\gamma(0)N_0} \leq h \leq \sqrt{\frac{4P^2_{\text{peak}}}{\gamma(0)N_0}}
\end{align*}$$

(31)

It can be shown that $T(h)$ should be non-increasing function of $h$. Therefore, for a finite set of rates, the problem becomes

$$\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{m} \frac{1}{T_i} \int_{h_i}^{h_{i+1}} f_h(h)dh \\
\text{subject to} & \quad \sum_{i=1}^{m} \frac{1}{\sqrt{T_i}A(T_i)} \int_{h_i}^{h_{i+1}} f_h(h)dh \leq \sqrt{\frac{4P^2_{\text{av}}}{\gamma(0)N_0}} \\
\text{and} & \quad \sqrt{\gamma(0)N_0} \leq h_i \leq \sqrt{\frac{4P^2_{\text{peak}}}{\gamma(0)N_0}}
\end{align*}$$

(32)

The results of the relative throughput for a system utilizing a photodiode with cutoff frequency of 50 MHz under various values of PAPR are shown in figure 6. As in the unlimited bandwidth case, joint adaptation of the power and symbol-rate significantly improves the performance.

V. CONCLUSION

Adaptive symbol-rate and adaptive power and symbol-rate transmission schemes for the free space optical channel are described and evaluated. Adaptive symbol-rate improved the
performance where the amount of gain depends on the strength of the channel scintillation and the number of utilized rates. It was also found that optimizing octave-spaced symbol rates can lead to very similar results compared to the all-optimized rates case when the number of rate is large. In the case of joint adaptation of power and symbol-rate, further gain was obtained which increases as the value of PAPR is increased.

The performance was also evaluated when a bandwidth limited detector is used. The bandlimited response of the receiver can reduce the obtainable gain depending on other system parameters such as the transmission power and the system noise.

Throughout the paper, the optical noise is considered to be zero to allow tractable analysis. However, a general model system noise.

The noise might also be effected by the bandlimited response of the channel scintillation and the number of utilized rates. A more general noise model is currently under investigation and will appear in future publications.

REFERENCES